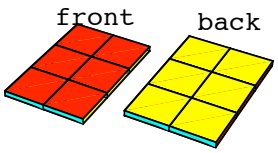
The back and the front puzzle.

-a flat version of Rubik's Cube-

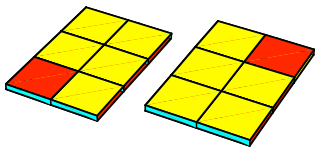
Ryohei Miyadera
Munetoshi Sakaguchi
Daisuke Minematsu
Ryota Kawazoe
Toshiro Miura
Kwansei Gakuin high school

1. Introduction.

The back and front puzzle is a flat version of **Rubik's Cube.** Alt
this game is very interesting in its own way. This game was first ir
front is colored with red and the back is colored with yellow.



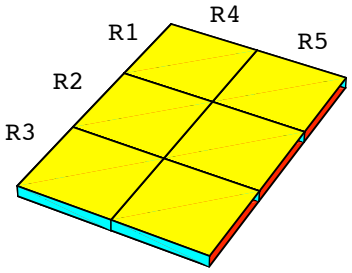
Shigeo Takagi treated **the back and front puzzle** as if it is a gam
arrangements in graph (1) can be seen identical, because you can get
We use a different approach. We fix our puzzle on a paper or scr
different ones, but our approach can lead to beautiful results that y
problem of graph theory, and found a very beautiful patterns in the p
first we are going to study the case of 3 rows and 2 columns. Later
the number of rows and columns turned out to be very important factor



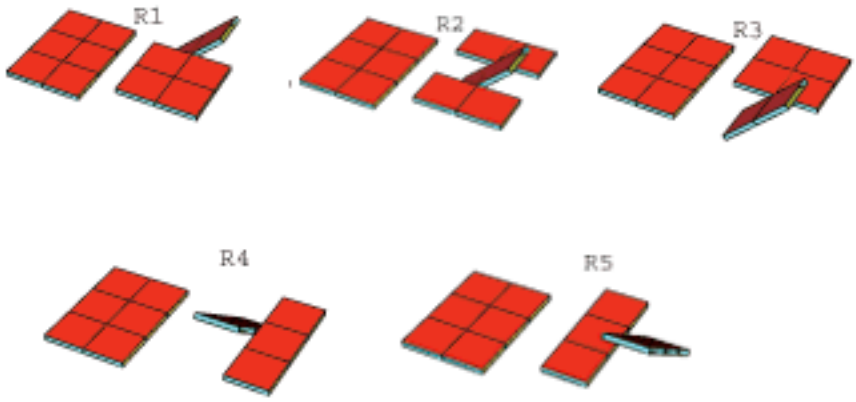
……Graph (1)

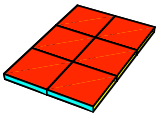
2. The case of 2×3.

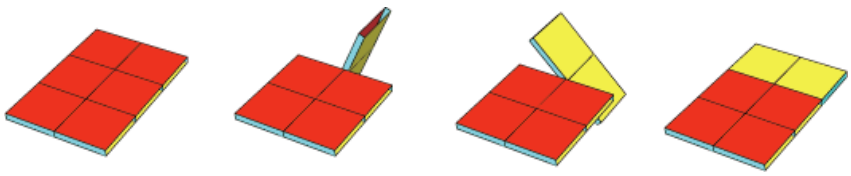
We only use five rotations for these arrangements. We name these rot



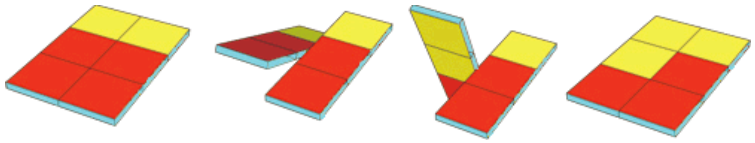
Example 1. The following pictures show you how these rotation occu

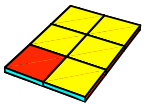


Example 2. (1) If you start with  and use R1, the

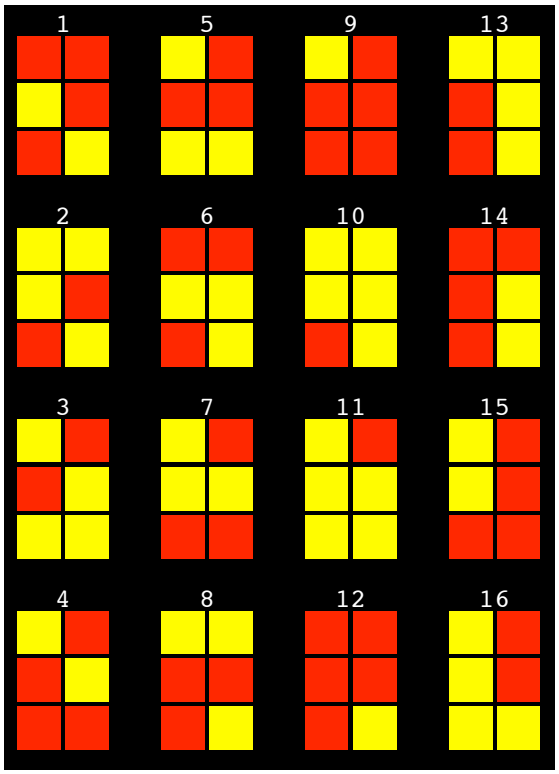


(2) After that if you use R4, then you get  . See the pi



Example 3. If you start with  , how many arrangements are the can use rotations as many times as you want.

Answer. You can get the arrangements in **Graph (2).**



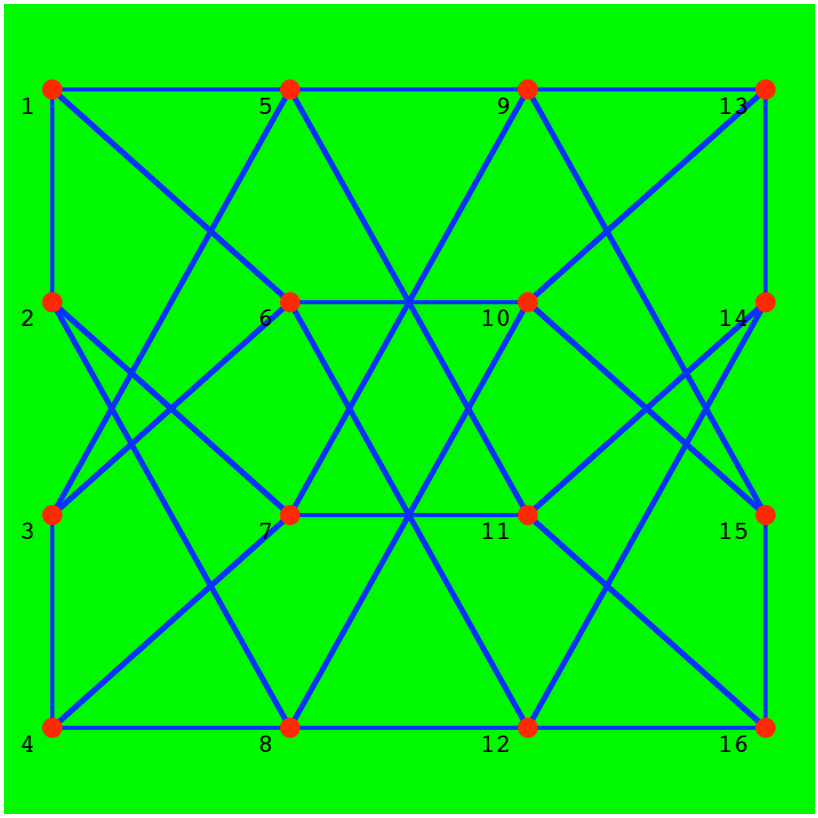
……**Graph (2)**

Example 4. It is a good way to use the theory of graph to study **the** vertex. If you can get an arrangement from another arrangement using

ing to these two arrangements with a blue line.

For example please look at the vertex 5 and 9. It is easy to see Therefore we connect them with a blue line. In the similar way we ca

When we made **Graph (3),** we chose vertexes with fewer lines and lo located vertexes with more lines in the middle of **Graph (3).**

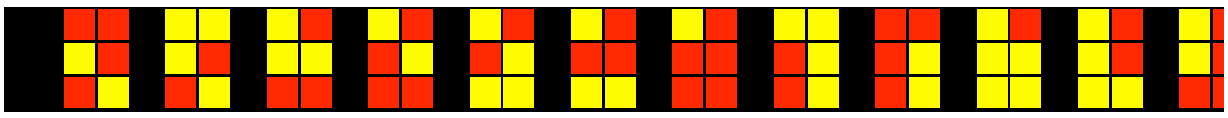


……**Graph (3)**

A Hamiltonian path is a path between two vertexes of a graph that path of the above graph?

Answer. A Hamiltonina path is {1,2,7,4,3,5,9,13,14,11,16,15,10,8,12,

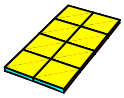
Perhaps it is easier to see the Hamiltonian path in the list of p to use are R1,R5,R4,R3,R5,R3,R5,R1,R4,R5,R3,R5,R2,R1 and R2.

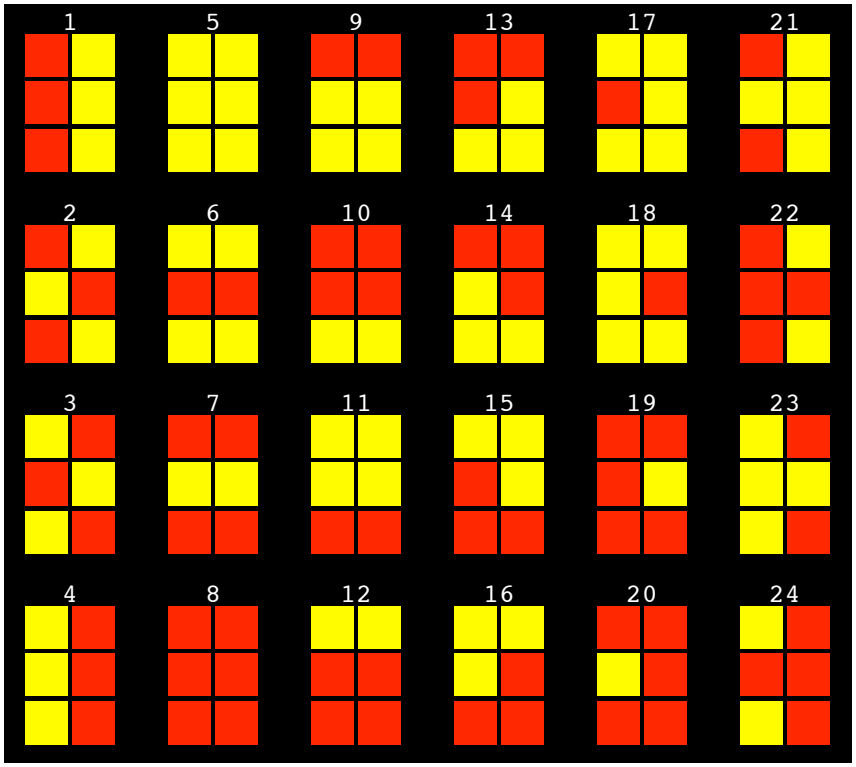


……**Graph (4)**

Problem 1. If you start with  , how many arrangements are ther
rotations as many times as you want.

Answer to problem 1.

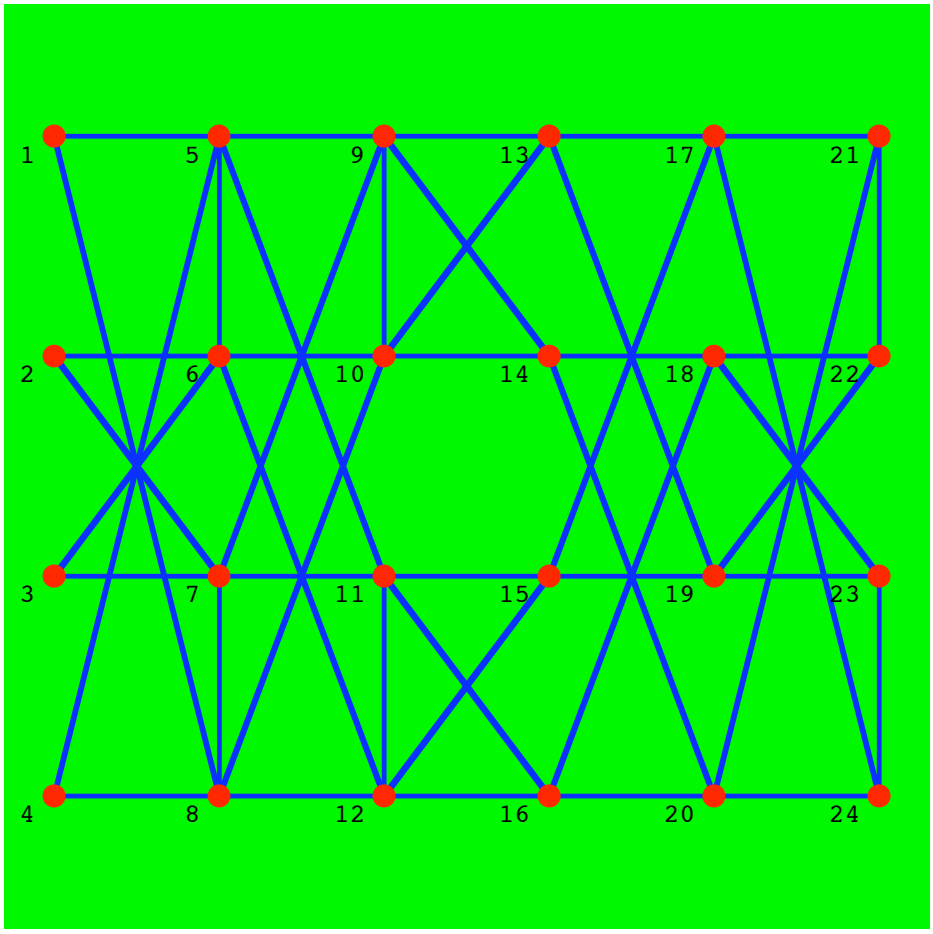
If you start with  , then there are 24 arrangement that you ca
check the answer once you get one. The order of arrangement in the f
best fit to the structure of **the Back and Front Puzzle.** You will see



……**Graph (5)**

Problem 2. Can you make a graph using **Graph (5)**? Can you find a Ham

Answer to problem 2.



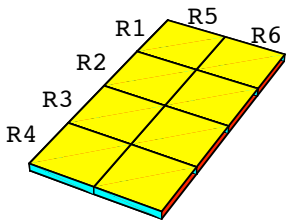
·····**Graph (6)**

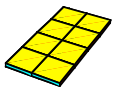
The following sequence is a Hamiltonian path. It is not difficult tc
{2,6,3,7,9,10,13,17,21,22,18,14,20,24,23,19,15,11,16,12,8,4,5,1}.

3. The case of 2×4.

In the previous section we studied the puzzle with 2 columns and 2 columns and 4 rows.

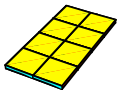
This time we have 6 rotations.



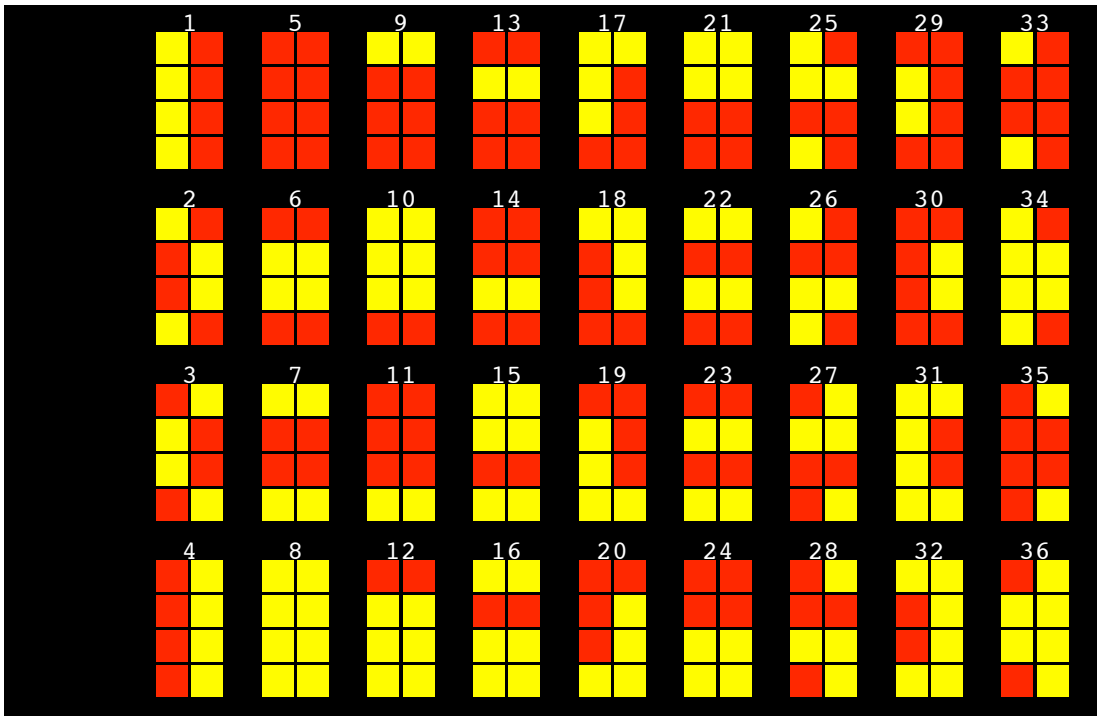
Example 5. (1) If you start with  and use R1, then

(2) After that if you use R5, then you get  .

Problem 3. If you start with ,how many arrangements are there that yc
as many times as you want.



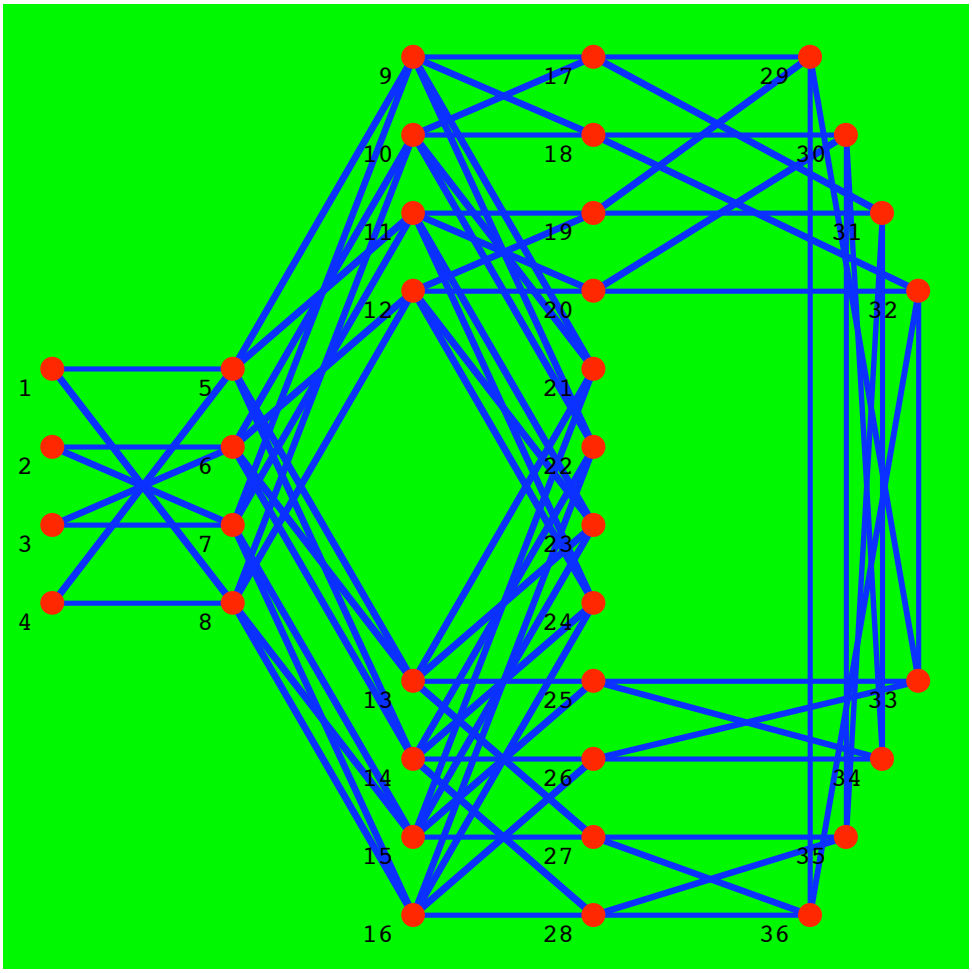
Answer to problem 3. It is not difficult to get all the 36 arrangeme
arrangements. If you get a good table, then it will make it easier t
It is better to locate vertexes with fewer lines in the first and the



……**Graph (7)**

Problem 4. (1) Can you make a beautiful graph using **Graph (7)**? (2) Can you find a Hamiltonian path?

Answer to problem 4 (1). It is not difficult to make a beautiful gra lot of lines in the middle part of graph, and vertexes with fewer lir



……**Graph (8)**

Answer to problem 4 (2). It is not easy to find a Hamiltonian path. the author of this article 6 hours to find a Hamiltonian path, but it found one. Perhaps there are many people who are a lot better than h *Mathematica*, it will take only a few minutes to find one.

{2,6,3,7,9,17,10,18,30,20,32,33,25,13,21,15,27,35,28,36,29,19,31,3 16,24,11,23,12,8,4,5,1} is a Hamiltonian path. Once you have a answe

Remark. We used *Mathematica* and the programs in S.Skiena and S. Pemm

References.

R. Miyadera, M. Sakaguchi, R. Kawagoe and T. Miura [1] E
Kwansei Gakuin Senior High School 1991.
T.Takagi [1] The back and front Puzzle, Suurikagaku 5 /
S.Skiena and S. Pemmaraju [1] Computational Discrete
Mathematics : Combinatorics and Graph Theory With *Mathe*
Cambridge Univ Pr.